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DYNAMICAL ORIGIN OF THE LORENTZIAN SIGNATURE OF SPACETIME [1]

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Abstract

It is suggested that not only the curvature, but also the signature of spacetime is subject to quantum fluctuations. A generalized D-dimensional spacetime metric of the form $g_{\mu\nu} = e^a_\mu \eta_{ab} e^b_\nu$ is introduced, where $\eta_{ab} = diag\{e^{i\theta}, 1, ..., 1\}$. The corresponding functional integral for quantized fields then interpolates from a Euclidean path integral in Euclidean space, at $\theta = 0$, to a Feynman path integral in Minkowski space, at $\theta = \pi$. Treating the phase $e^{i\theta}$ as just another quantized field, the signature of spacetime is determined dynamically by its expectation value. The complex-valued effective potential $V(\theta)$ for the phase field, induced by massless fields at one-loop, is considered. It is argued that $Re[V(\theta)]$ is minimized and $Im[V(\theta)]$ is stationary, uniquely in $D = 4$ dimensions, at $\theta = \pi$, which suggests a dynamical origin for the Lorentzian signature of spacetime.

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Spacetime curvature is a dynamical object in gravity theory; spacetime signature is not. With few exceptions [1, 2], the Lorentzian signature of the physical spacetime metric is simply taken as given and non-dynamical. Lorentzian signature can be enforced in quantum gravity by introducing tetrads e^a_μ, so that

$$g_{\mu\nu} = e^a_\mu \eta_{ab} e^b_\nu \tag{1}$$

is the spacetime metric and

$$\eta_{ab} = diag\{-1, 1, 1, 1\} \tag{2}$$

is the local frame Minkowski metric. The problem in relativistic quantum theory is then to evaluate Feynman path integrals of the form

$$Z_F = \int d\mu(e, \phi, \psi, \overline{\psi}) \exp\left[-i \int d^4x \sqrt{-g}\mathcal{L}\right] \tag{3}$$

with η_{ab} fixed, where $d\mu(e, \phi, \psi, \overline{\psi})$ is the integration measure for the tetrads, and other bosonic (ϕ) and fermionic ($\psi, \overline{\psi}$) fields.

For technical reasons, however, one often considers instead the Euclidean path integral

$$Z_E = \int d\mu(e, \phi, \psi, \overline{\psi}) \exp\left[-\int d^4x \sqrt{g}\mathcal{L}\right] \tag{4}$$

where this time

$$\eta_{ab} = diag\{1, 1, 1, 1\} \tag{5}$$

Comparing the Feynman and Euclidean path integrals, it is easy to write down a slightly more general path integral which interpolates between them, namely (in D dimensions)

$$Z = \int d\mu(e, \phi, \psi, \overline{\psi}) \exp\left[-\int d^Dx \sqrt{g}\mathcal{L}\right] \tag{6}$$

where

$$\eta_{ab} = diag\{e^{i\theta}, 1, ..., 1\} \tag{7}$$

and we obtain the Euclidean theory for $\theta = 0$, and the Feynman theory for $\theta = \pi$, with the correct $i\epsilon$ prescription for propagators supplied automatically as $\theta \to \pi$. The theory at $\theta \to -\pi$ converts to the Feynman theory under a time inversion.

It should be stressed that for quantum gravity, the continuation from the Minkowski to the Euclidean theory is really a continuation in signature η_{ab}, rather than just a rotation $t \to e^{i\theta}t$ of the time coordinate. Without continuation in the signature, the local frame invariance of general relativity would be $O(3,1)$ in both Minkowski *and* Euclidean space, instead of changing from $O(3,1)$ to $O(4)$ in Euclidean space. Moreover, expectation values of certain diffeomorphism invariant quantities (such as $< \int \sqrt{g}R^p >$) have no dependence whatever on time intervals, and the difference between expectation values in the Euclidean and Minkowski theories resides entirely in their dependence on the determinant of the signature $det(\eta)$.

The introduction of a generalized signature (7) then suggests the possibility of viewing the phase factor $\exp(i\theta)$ as a dynamical field in its own right. In that case, the signature of spacetime will be determined dynamically, by the expectation value of this phase field. To compute the signature of spacetime, we need to compute the effective potential $V(\theta)$ for the $\exp[i\theta(x)]$ phase field, which is generated after integrating out all other fields - matter, gauge, and tetrad. Apart from the form of the Langrangian, the final answer also calls for some assumptions about the θ-dependence of the integration measure $d\mu(e, \phi, \psi, \overline{\psi})$, which is otherwise taken proportional to the (real-valued) DeWitt measure. In this letter I will just point out the consequences of the following simple assumptions about the measure, which fix this θ-dependence:

1. For free fields of mass m, the contributions to Z in eq. (6) from each (propagating) bosonic degree of freedom are equal, and inverse to the contributions from each fermionic degree of freedom. Thus, e.g., $Z = 1$ at any θ for a supersymmetric combination of free fields.

2. The integration measure for scalar fields is given by the real-valued, invariant volume measure (DeWitt measure) in superspace $d\mu(\phi) = D\phi\sqrt{|G|}$, where G is the determinant of the scalar field supermetric $G(x, y) = \sqrt{g}\delta(x - y)$.

Consider the one-loop contribution to $V(\theta)$ due to integration over a massless scalar field ϕ in a flat background $e^a_\mu = \delta^a_\mu$. Denoting this contribution by $V_0(\theta)$, we have

$$\exp[-\int d^D x V_0(\theta)] = det^{-\frac{1}{2}}[-\sqrt{\eta}\eta^{ab}\partial_a\partial_b] \tag{8}$$

and proper-time regulation of the determinant gives

$$
\begin{aligned}
V_0(\theta) &= -\frac{1}{2}\int_{1/\Lambda^2}^{\infty}\frac{ds}{s}\int\frac{d^Dp}{(2\pi)^D}\ \exp[-s(e^{-i\theta/2}p_0^2+e^{i\theta/2}\vec{p}^{\,2})] \\
&= -\frac{\Lambda^D}{D(4\pi)^{D/2}}\exp[-i(D-2)\theta/4]
\end{aligned} \tag{9}
$$

where Λ is a momentum cutoff which, given the non-renormalizability of gravity, presumably exists at the Planck scale. The p-integration in (9) is only well-defined for θ in the range $[-\pi,\pi]$, which is related to the fact that $Re[\sqrt{g}\mathcal{L}_\phi]$ for a scalar field is only bounded from below for $|\theta|\le\pi$.[3]

For higher-spin massless fields, it is straightforward to verify that, up to some extra factors of $det^p(\eta)$, the one-loop contribution from each massless bosonic field is given by $det^{-\frac{1}{2}}(-\sqrt{\eta}\eta^{ab}\partial_a\partial_b)$ raised to the power n_B (the number of propagating degrees of freedom),[4] while for spinor fields it is this quantity raised to the power $-n_F$ (no. of fermionic degrees of freedom $\times -1$). In a curved space-time background, the only difference is that the argument of the root determinant changes to $-\sqrt{g}g^{\mu\nu}\partial_\mu\partial_\nu$. Any additional factors of $det^p(\eta)$ that arise in the integration are, by assumption, cancelled by a corresponding factor in the measure. Therefore, taking all massless fields into account, we have

$$
V(\theta)=(n_F-n_B)\frac{\Lambda^D}{D(4\pi)^{D/2}}\exp[-i(D-2)\theta/4] \tag{10}
$$

as the one-loop effective potential for the phase field.

$V(\theta)$ is complex-valued. To determine $< e^{i\theta} >$ at one-loop level, we look for a value of θ in the range $[-\pi,\pi]$ such that: i) $Re[V]$ is minimized; and ii) $Im[V]$ is stationary. If these two conditions are not satisfied for the same value of θ, then there may be large quantum fluctuations in the signature. From (10), the requirements are seen to be:

$$
\left.\begin{aligned}
\cos\left[(D-2)\theta/4\right]=0 \\
\min\left[Re[V(\theta)]\right]=0
\end{aligned}\right\}\quad \theta\in[-\pi,\pi] \tag{11}
$$

[3] For the same reason, the usual Wick rotation from Minkowski to Euclidean time must be taken as $t\to -it$ rather than $t\to +it$.

[4] E.g. $n_B=(D-2)$ degrees of freedom for massless vector fields; $n_B=D(D-3)/2$ for the graviton field.

The first condition is just the requirement that $Im[V]$ is stationary. Then $Re[V] = 0$ at the stationary point of $Im[V]$, and the second condition is the requirement that this is also the minimum of $Re[V]$. The constraint on the range of θ is needed, as mentioned above, to ensure the existence of the functional integral over the scalar field.

The set of conditions (11) cannot be solved in arbitrary dimensions; in fact, for $n_F < n_B$ there is no solution in any dimension. This is because, for $n_F < n_B$, $\min[Re[V(\theta)]] = V(0) \neq 0$. For $n_F > n_B$ there is only one solution, namely, $D = 4$ dimensions and $\theta = \pm\pi$. This can be seen from the fact that if $(D-2)\pi/4 > \pi/2$, then $\min[Re[V(\theta)]] < 0$, and similarly if $(D-2)\pi/4 < \pi/2$, the minimum of the real part is greater than zero. Only at $(D-2)\pi/4 = \pi/2$, i.e. at $D = 4$, can all conditions be satisfied, and this is just at $\theta = \pm\pi$. Remarkably, *it appears that $V(\theta)$ uniquely singles out both Lorentzian signature and the observed dimensionality of spacetime.*

In this argument I have neglected the fact that the gravitational action is unbounded from below for any θ, due to the "wrong-sign" of the kinetic term for the conformal factor. The one-loop contribution from the gravitational field to $V(\theta)$ is therefore ill-defined. There are a number of approaches to defining the Euclidean path integral for gravity, such as stochastic stabilization [3], contour rotation [4], and other ideas [5]. But whichever prescription is used, so long as the eigenvalues of the graviton kinetic operator are proportional to those of the Klein-Gordon operator $\eta^{ab}\partial_a\partial_b$, the above conclusion concerning signature and dimension is unaffected.

The possibility that spacetime signature might fluctuate raises many questions. Conceivably such fluctuations would be important at the Planck scale, or relevant to the last stages of gravitational collapse. Supersymmetry is also a concern since, at the one-loop level, Lorentzian signature arises at D=4 only if $n_F > n_B$.[5] If Nature is supersymmetric ($n_F = n_B$), then the expectation value of the signature presumably depends on the details of the supersymmetry breaking. This issue requires further study.

The Euclidean $\rightarrow$ Minkowski interpolation takes us from a real measure, $\exp(-S)$ to a complex measure, i.e. from statistical mechanics to quantum mechanics. An obvious further generalization would be to have η_{ab} interpolate between all possible signatures, with arbitrary numbers of $\pm$ signs corresponding to arbitrary numbers of time-like coordinates. However, quantum theory with more than one time variable

[5] In connection with the role of massless spinors, I have learned that Nielsen [6] has recently arrived at the same conclusion regarding signature and dimension based on properties of chiral fermions, although his reasoning is quite different from the argument presented here.

has not, to my knowledge, been formulated, and it is not obvious what the correct measure for the corresponding functional integral should be. One could speculate that interpolation to quantum mechanics with multiple time-like coordinates might call for some generalization of complex numbers, such as the quaternions or octonions; but at present it is unclear if multi-time generalizations of quantum theory can be formulated consistently.

To summarize, if the signature of the metric is a dynamical quantity, then its expectation value is determined by quantum effects. In this letter I have discussed the one-loop effective potential for the generalized signature (7) induced by massless fields, given certain assumptions about the functional integration measure. It is found that if the number of fermionic degrees of freedom exceeds the number of bosonic degrees of freedom, then the real part of the effective potential is minimized (and the imaginary part is stationary) only for Lorentzian signature, and only in D=4 dimensions.

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